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Axial dispersion model for solid flow in liquid suspension in system of two mixers in total recycle

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Abstract

The measurement of residence time distribution of solid particles in solid–liquid suspension is experimentally difficult. However, the *twin system approach* is particularly suited for the assessment of particle RTD in flow systems as it allows overcoming some of the usual difficulties generally encountered in this kind of measurement. Twin system consists of two vessels and external piping in total recycle. Experimental results from this system can be evaluated using Z-transforms to derive particle RTD for subsequent testing of alternative flow models. Recently, the axial dispersion model was applied using the "advection diffusion equation" (sometimes called the "diffusion with bulk flow equation") derived thereof, which was solved numerically. This contribution presents an analytical solution of analogous equations, which enables direct and precise evaluations of this problem.

Keywords: Residence time distribution; Solid–liquid flow; Axial dispersion model; Mixing

1. Introduction

Operations in which an important role is played by flow of solid suspensions in a liquid phase, often appear in industrial applications. Because the velocity of solid particles is generally different from the liquid velocity, the RTD of the former is bound to differ from that of the latter. As a consequence, it is important, e.g. in the case of chemical reactors, to separately measure the residence time distribution of solid particles in the apparatus, which is often vital for accurate conversion calculations [1]. An easy to measure quantity in a real vessel is the mean particle residence time, which, on the basis of the relevant theorem, is given by the following relation:

$$t_{\mathrm{id}} = \frac{V}{Q}, \tag{1}$$

where V is the volume of particles held-up in the vessel and Q is the volumetric flow rate of solid particles. Unfortunately, this quantity cannot provide information on important real flow features such as by-passing, short-circuit, etc. These can be found only by suitable experimentation, but this turns out to be much more complex than for liquid phases.

One of the methods to achieve such measurements is the *twin system approach*, which was suggested by Brucato and Rizzuti [2] and is described, together with the results obtained thereof, in several other publications [3–5]. The twin system consists of two identical stirred reactors connected by external piping fitted with a pump and a detector of traced particles concentration. The subject of interest is obtaining the mentioned characteristics of residence time in stirred reactors.

At the beginning, the two systems are filled with known and equal amounts of both traced and untraced phase and then operated in a "self recycle configuration" (SRC) as depicted in Fig. 1a) with flow rates, agitation speeds and any other operational variable identical for the two systems. The detector is placed approximately in the middle, between the stirred vessels. After homogenization is achieved, the two halves are connected at the middle, such that both reactors are now integrated in series and the whole system operates in total recycle. The advantage of this arrangement is the reality that to evaluate the differential distribution function, tracer concentration measurement at one point of the system only is required, as arises from Eq. (2) of

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Nomenclature

a	dispersion number, reciprocal Peclet number
b	reciprocal mean residence time, dispersion model parameter (s^{-1})
c	concentration ($kg\,m^{-3}$)
c_1	dimension concentration constant ($kg\,m^{-3}$)
C	dimensionless concentration
D_a	axial dispersion coefficient ($m^2\,s^{-1}$)
$E(t)$	differential distribution function (s^{-1})
$f_0(y)$	dimensionless initial distribution of concentration
L	length of the system (m)
N	agitator speed (min^{-1})
Q	volumetric flow rate ($m^3\,s^{-1}$)
t	time (s)
t_{id}	theoretical mean residence time (s)
T	mean residence time (s)
v	mean velocity of the solid particles ($m\,s^{-1}$)
V	volume (m^3)
x	spatial co-ordinate (m)
y	dimensionless spatial co-ordinate (m)

Greek letters

α_n	nth root of transcendent Eq. (12)
$\varphi_0(\tau)$	dimensionless input function, stimulus
σ^2	variance (s^2)

Subscripts

e	refers to external pipes
i	general index
m	refers to a mixer
n	refers to nth root of Eq. (15)
p	refers to sum of two mixers and external pipes
w	refers to all system

contribution [5]:

$$c(t) = \int_0^t [1 - c(t - t')]E_w(t')\,dt' \quad (2)$$

In this integral equation, symbol $c(t)$ denotes measured, normalized tracer concentration and $E_w(t)$ is the sought after differential distribution function of particle residence time in one half.

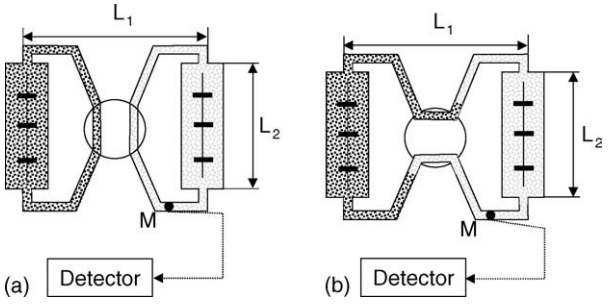

Fig. 1. Scheme of twin system approach: (a) self recycle configuration (SRC) and (b) cross mixed configuration (CRC).

A possible shortcoming of this arrangement with respect to evaluation is the fact that, due to technical reasons, the volume of external piping cannot be minimized up to becoming negligible with respect to the volume of stirred vessels. In order to exclude the external piping effect, the standard experiment was carried out, in which the reactors were exceptionally bypassed and the experiment reproduced with adequate amount of particles. From the evaluation of such modified experiment, the unknown function is obtained—the differential distribution function of external piping $E_e(t)$. The desired function $E_m(t)$, i.e. differential distribution function of particle residence time in stirred vessel is found by using another integral equation (convolution integral):

$$E_w(t) = \int_0^t E_m(t')E_e(t - t')\,dt' \quad (3)$$

The solution of Eqs. (2) and (3) was evaluated using the discrete Z-transform, which leads to simple numerical procedure [3–5], which is however, somewhat imprecise, just like any other deconvolution procedure (e.g., see [1]).

The recorded response of twin system was evaluated on the basis of combined model concepts. The twin system, i.e. external piping and vessel was considered as a combination of piston flow and cascade of ideal mixers (tank in series fractional tubularity model) [6], a combination of two ideal mixers cascades [6] and tanks in series with backflow [5]. Recently, the efforts are underway to describe the response by axial dispersion model [7], i.e. numerical solution of the "advection diffusion equation" (see below Eq. (4)).

In this contribution, we also apply this equation, with the difference being that we present its analytical solution (see also [8,9]) for the initial and boundary conditions relevant to the situation in the system.

It is assumed that the obtained shape of the function applies to the mixers, to the external piping as well as for the whole system, differing only in the values of the two parameters, v and D_a in Eq. (4) or parameters derived thereof. These values were obtained by non-linear regression method, where differences between experimental and theoretical responses are minimized. In this way, use of the Z-transform and numerical solutions of the partial differential equation, which may be a source of error, is thus avoided.

2. Theoretical

The twin system scheme is reported in Fig. 1. The situation before the start of the experiment is called the *self recycle configuration* (SRC; Fig. 1a). Branded particles – traced particles – are placed in the left half of the twin system. The concentration is measured at point M. At time $t < 0$, it is equal to zero. L_1 is half a length of the external piping and L_2 is the height of the mixer. During experiment the whole system is connected *cross mixed configuration* (CMC; Fig. 1b).

We assumed that the system can be described by the following equation:

$$\frac{\partial c}{\partial t} + v\frac{\partial c}{\partial x} - D_a\frac{\partial^2 c}{\partial x^2} = 0 \quad (4)$$

where x denotes the spatial co-ordinate, v the mean velocity of particle, D_a the effective axial dispersion coefficient and c is the tracer concentration.

Danckwerts boundary conditions are assumed for Eq. (4):

$$vc - D_a \left.\frac{\partial c}{\partial x}\right|_{x=0+} = vc_1\varphi_0(t)|_{x=0-} \tag{5}$$

$$\frac{\partial c}{\partial x} = 0|_{x=L} \tag{6}$$

and the initial condition as follows:

$$c(x, 0) = c_1 f_0(x) \tag{7}$$

where constant c_1 has the dimensions of concentration and thus $f_0(x)$, which determines initial distribution of particles in system and $\varphi_0(t)$, which determines time dependence of concentration change on the beginning of system, are dimensionless. The dimensionless concentration and longitudinal co-ordinate are defined:

$$C = \frac{c}{c_1}; \quad y = \frac{x}{L_i} \tag{8}$$

It will be assumed, further on, that the system of Eq. (4)–(8) is valid for parts of the system as well as for the whole system and that for external piping L_e, for mixer L_m and for whole system L_w:

$$L_e = 2L_1, \quad L_m = L_2, \quad L_w = 2(L_1 + L_2) = L_e + 2L_m \tag{9}$$

where L_e is total length of external piping, L_m the height of one mixer and L_w is the length of whole system. Other parameters b_i and a_i are introduced into Eq. (4):

$$b_i = \frac{v_i}{L_i} = \frac{1}{T_{ti}} \quad [i = e, m, w] \tag{10}$$

$$a_i = \frac{D_{ai}}{v_i L_i} = \frac{1}{Pe_i} \quad [i = e, m, w] \tag{11}$$

where b_i is reciprocal value of mean residence time in the appropriate part of the system and a_i is the reciprocal value of Peclet number.

Eq. (4) can therefore be rewritten as:

$$\frac{\partial C}{b_i \partial t} + \frac{\partial C}{\partial y} - a_i \frac{\partial^2 C}{\partial y^2} = 0 \quad [C = C(y,\ t),\ 0 < y < 1, \quad 0 < t] \tag{12}$$

The boundary conditions (5), (6), and initial condition (7) are transformed in a similar way. Fourier's method of solution is used, where the unknown function of two variables is substituted by a product of two functions (each a function of only one co-ordinate) $C(y, t) = P(y)Q(t)$. The general solution in dimensionless form is presented by Kudrna et al. [11]:

$$C_i(y, t) = 2\,e^{y/(2a_i)} \sum_{n=1}^{\infty} \frac{a_i}{h_{ni} + 1} P(y, \alpha_{ni}) Q(b_i t, \alpha_{ni}) \times \left[\int_0^1 f_0(y')\, e^{-y'/(2a_i)} P(y', \alpha_{ni})\, dy' + \alpha_{ni} \int_0^{b_i t} \frac{\varphi_0(\tau)}{Q(\tau, \alpha_{ni})}\, d\tau \right] \quad [i = e, m, w] \tag{13}$$

where:

$$\left.\begin{aligned} P(y, \alpha_{ni}) &= \alpha_{ni} \cos(\alpha_{ni} y) + \left(\frac{1}{2a_i}\right) \sin(\alpha_{ni} y) \\ Q(b_i t, \alpha_{ni}) &= \exp(-h_{ni} b_i t) \\ h_{ni} &= \frac{4a_i^2 \alpha_{ni}^2 + 1}{4a_i} \end{aligned}\right\} \quad [i = e, m, w] \tag{14}$$

and α_n are the positive roots of the transcendental equation:

$$\tan(\alpha_{ni}) = \frac{4a_i \alpha_{ni}}{4a_i^2 \alpha_{ni}^2 - 1} \quad [i = e, m, w] \tag{15}$$

The differential distribution function of particles both for the whole system and also for parts of it [i = e, m, w] can be easy derived from the general Eq. (13). Time dependence of the dimensionless concentration of particles at the end point of the relevant system part ($y = 1$) must be recorded with assumption that all particles are present at the point of input ($y = 0$) at the initial time:

$$E_i(t) = C_i(1, t) b_i \quad [f_0(y) = 0;\ \varphi_0(t) = \delta(b_i t)] \tag{16}$$

where δ represents the Dirac impulse. It was specially demonstrated (see paper [10,11] or more detailed in [12]) that the first equation of Eq. (14) can be simplified to:

$$P(1, \alpha_{ni}) = (-1)^{n-1} \alpha_{ni} \tag{17}$$

After substitution of $f_0(y)$, $\varphi_0(t)$, which are defined in Eq. (16), into the Eq. (13) and rearrangement we obtain:

$$E_i(t) \equiv E_i(t, a_i, b_i) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} 2a_i\, e^{1/(2a_i)} \alpha_{ni}^2}{1 + h_{ni}} e^{-h_{ni} b_i t} b_i = \sum_{n=1}^{\infty} q_{ni}\, e^{-h_{ni} b_i t} b_i \quad [i = e, m, w] \tag{18}$$

The last equation can be considered as definition of the term q_{ni}, which was introduced for brevity's sake.

However, the arrangement under study does not make it possible to determine directly the function $E_i(t)$, thus, the system is in total recycle, one half is filled by traced particles and the second half is not (see Fig. 2) at the beginning. The end of the system ($y = 1$) is considered as point M (as it is shown in Fig. 2).

The initial and boundary conditions are defined with respect to Fig. 2. The functions $f_0(y)$, $\varphi_0(\tau)$ from Eq. (19) are substituted

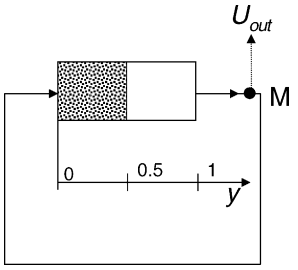

Fig. 2. Dispersion model conception of twin system approach.

into the Eq. (13):

$$f_0(y) = \begin{cases} 1 & \left[0 \le y \le \frac{1}{2}\right] \\ 0 & \left[\frac{1}{2} \le y \le 1\right] \end{cases} \tag{19}$$

$$\varphi_0(t) = C_i(1, t) \equiv C_{1i}(t) \tag{20}$$

The first integral in Eq. (13) is readily solved when Eq. (15) is utilized:

$$\int_0^{1/2} \mathrm{e}^{-y'/(2a_i)} P(y', \alpha_{ni})\,\mathrm{d}y' = \frac{\alpha_{ni}}{h_{ni}}\left(1 - \frac{\exp(1/(4a_i))}{2\cos(\alpha_{ni}/2)}\right) = \frac{\alpha_{ni} g_{ni}}{h_{ni}} \quad [i = \mathrm{e}, \mathrm{w}] \tag{21}$$

Eq. (21) is simultaneously the definition of the term g_{ni}.

After substitution of these relations into Eq. (13) and concurrently the sequence replacement of summation and integration we obtain the final relation for the calculation of total recycle response:

$$C_{1i}(t) = \int_0^t C_{1i}(t') \sum_{n=1}^{\infty} q_{ni}\, \mathrm{e}^{-h_{ni} b_i (t-t')} b_i\,\mathrm{d}t' + \sum_{n=1}^{\infty} q_{ni} \frac{g_{ni}}{h_{ni}}\, \mathrm{e}^{-h_{ni} b_i t} \quad [i = \mathrm{e}, \mathrm{w}] \tag{22}$$

Eq. (22) is an integral equation of the Volterra type, which for given values of parameters a_i, b_i makes it possible to calculate theoretical values of concentration at the point of detection. Optimal values of parameters can be found by comparison of measured and calculated responses from Eq. (22) for external piping and the whole system (see Eq. (24)). Obtained optimum values of a_i and b_i for the external piping (i = e) as well as for the whole system (i = w) are substituted into Eq. (18) and we directly compute the differential distribution function of residence time of particles for this arrangement. To extract the differential distribution function of mixer from system the relation (23), similar to Eq. (3) is used. Owing to the existence of two mixers in the system the convolution ought to be undertaken twice, i.e.:

$$E_{\mathrm{w}}^{\mathrm{con}}(t, a_{\mathrm{w}}, b_{\mathrm{w}}) = \int_0^t E_{\mathrm{m}}(t - t') \left[\int_0^{t'} E_{\mathrm{m}}(t' - t'') E_{\mathrm{e}}(t'')\,\mathrm{d}t''\right] \mathrm{d}t' \tag{23}$$

Values of mixer parameters can then be optimized by comparison of the differential distribution function of the whole system (i = w) from Eq. (18) and the calculated differential distribution function of the whole system from Eq. (23). The resulting differential distribution function of mixer is calculated from optimized parameters a_{m} and b_{m} by Eq. (18).

3. Experimental

The twin system is used to obtain solid residence time distribution in a two phase stirred reactor. The twin system approach (TSA) technique, formerly introduced by Brucato and Rizzuti [2] may be regarded as a convenient alternative to the conventional pulse and step disturbance techniques. It is particularly suited for the assessment of particle RTD in flow systems, as it allows overcoming some of the experimental difficulties (e.g. instability of flow). As a matter of fact, the TSA technique has already been successfully applied to the case of particle RTD in a single-impeller stirred vessel [3] and in the system here investigated [5,7]. In plant practice (i.e. industrial operations), the use of radioactive tracer technique has been well established [1].

Experimental set up is shown in the Fig. 1. The system investigated was already described in ref. [5] and it consisted of a cylindrical stirred vessel, 100 mm in diameter, 300 mm in height, provided with four standard removable baffles and stirred by three equally spaced Rushton turbines of 50 mm diameter. The first impeller was placed at $T/2$ from the bottom, while the other two were spaced at a distance equal to T from each other. The vessel bottom had a semi-spherical shape, with the slurry outlet placed in the middle, in order to help preventing particle fillet build-up. The slurry inlet was through a hole located at the vessel top cover. The measured volume of the vessel was 2360 cm^3. The volume involved in the portion of apparatus external with respect to the investigated vessel was 630 cm^3 for each of the twin system vessels, thus accounting for about 21% of the total volume.

Experimental arrangement, method of detection and regeneration of particles is well described in papers [2–7]. The experimental apparatus involved two identical stirred vessels, two magnetic flow-meters, two centrifugal pumps, a speed controlled DC motor that drove both stirrer shafts through suitable belts and pulleys and a continuous on-line detector to measure the traced particle fraction connected to a data acquisition system. In order to continuously detect the fraction of traced particles, the inexpensive photo-resistor detector described in [4] was adopted. External piping was almost wholly made of transparent PVC (i.d. = 14 mm), so that it was possible to check that particle deposition in the external circuitry was avoided. Due to its volume size, the external circuitry contribution is certainly significant in the present case. This rules out the possibility of using the previously quoted, comprehensively simplified approach and implies the need of properly accounting for the contribution of the external tubing during data analysis.

Different agitation speeds (400, 600, 800, 1000 and 1400 rpm) were explored while the total (liquid + particles) external volumetric flow rate was always maintained at 6 l/min. One experiment with removed baffles was made at agitation

speed 600 rpm. Tap water at room temperature was used as the liquid phase for all experimental runs.

Accurately sieved silica particles (0.180–0.212 mm) were used as the solid phase in all runs. The mass of solid particles introduced in each twin vessel was always 0.175 kg.

4. Results and discussion

Traced particle concentrations were measured at equidistant time intervals ($\Delta t = 0.5$ s) and recalculated according to the method described in [5,7]. The theoretical response $C_{1t}^{cal}(t, a_i, b_i)$ was calculated from Eq. (22) by trapezoidal numerical integration for aptly chosen values of parameters a_i and b_i. The choice of values for the parameters was made by graphically comparing the theoretical response to experimental response. Optimized parameters of the whole system and external piping differential distribution functions were then calculated with the help of non-linear regression, where function $S(a_i, b_i)$, defined by Eq. (24) was minimized. The examples of responses are shown in Figs. 3 and 4. The response of external piping with by-passed mixers is depicted in Fig. 3. The experimental responses of the

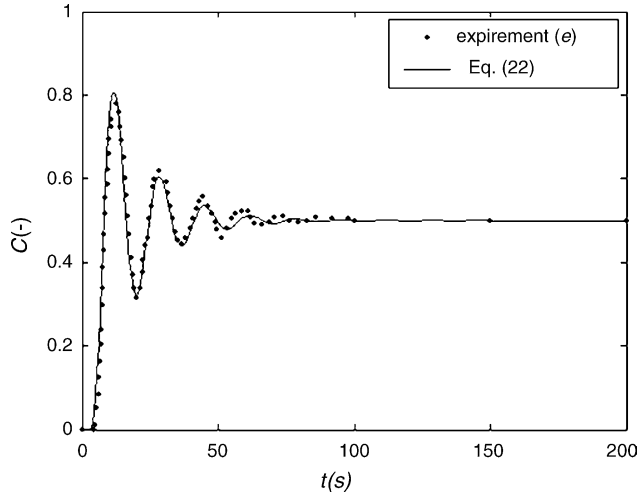

Fig. 3. Experimental and calculated response of external piping obtained by dispersion model; $V = 1.26$ l, $Q = 6$ l/min.

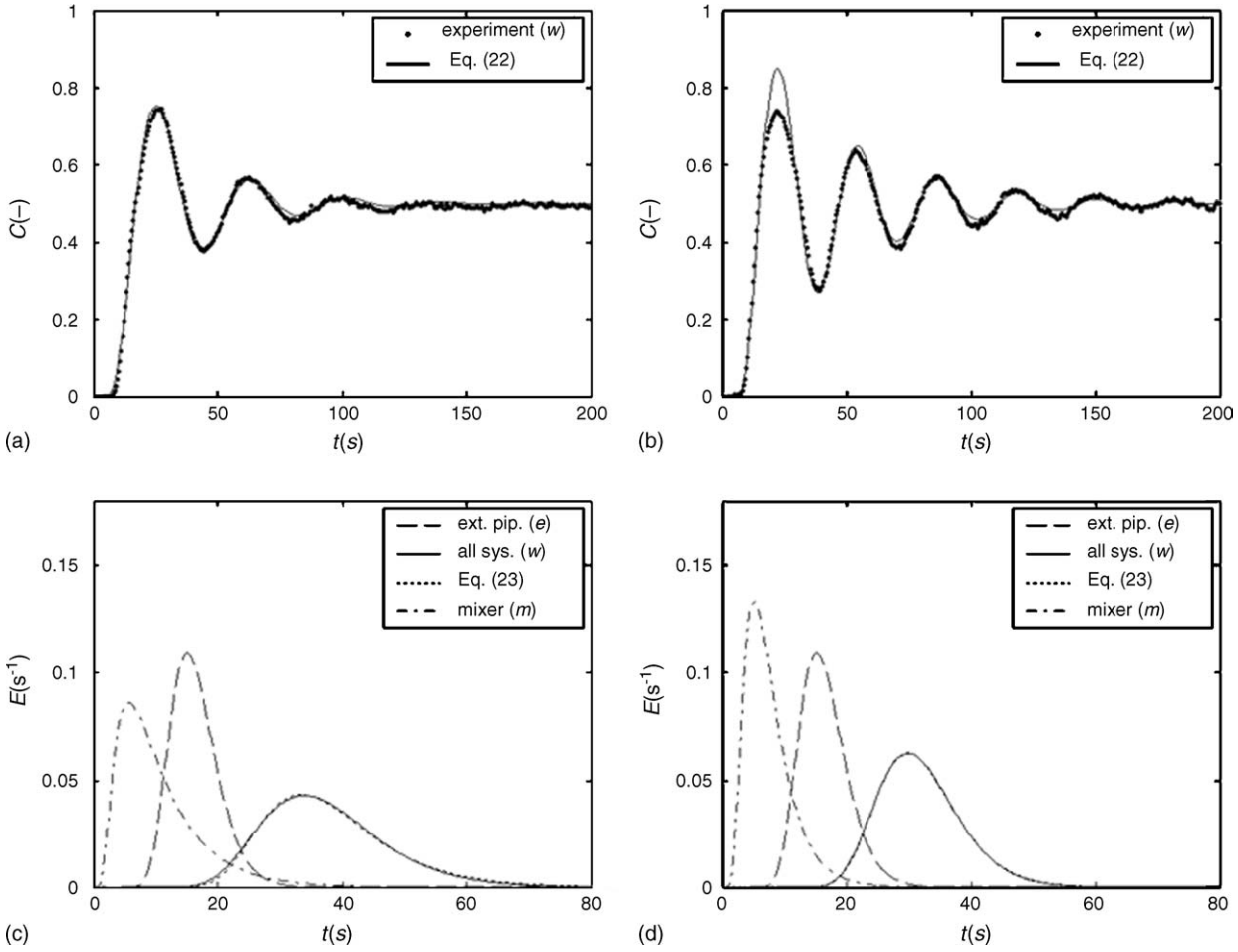

Fig. 4. Evaluation experiment, $V = 2.74$ l, $Q_L = 6$ l/min: (a and b) experimental and calculated response of external piping and two mixers together obtained by dispersion model—(a) $N = 400$ rpm, with baffles, (b) $N = 600$ rpm, without baffles; (c and d) differential distribution function: external piping, mixer, all system—(c) $N = 400$ rpm, with baffles, (d) $N = 600$ rpm, without baffles.

system with external piping and mixers are highlighted in Fig. 4a and b. In the case of Fig. 4b, the baffles were removed from the vessels.

$$S(a_i, b_i) = \sum_{j=1}^{N} (C_{1i}^{\exp}(t_j) - C_{1i}^{\mathrm{cal}}(t_j, a_i, b_i))^2 \quad [i = \mathrm{e}, \mathrm{w}] \qquad (24)$$

Infinite summations in Eq. (20) were replaced by finite summation with 300 terms. We obtained optimal values of parameters a_e and b_e, in the case of bypassed mixers as well as a_w and b_w for the whole system. These values are substituted in Eq. (23), just as were the aptly chosen values of a_m and b_m. Non-linear regression is used again to obtain optimal values of these parameters, where $S(a_m, b_m)$ was minimized:

$$S(a_m, b_m) = \sum_{j=1}^{N} (E_w(t, a_w, b_w) - E_w^{\mathrm{con}}(t, a_m, b_m))^2 \qquad (25)$$

The first term on the right hand side is calculated from Eq. (18) for all system [i = w] using parameters found from Eq. (22). The second term is calculated by use of a two-fold trapezoid numerical integration of Eq. (23). The differential distribution functions of external piping, of the mixer and of the whole system are shown in Fig. 4c and d, where Fig. 4d relates to the situation without baffles. The shape of the differential distribution function of the mixer without baffles is sharper. The differential distribution functions of the mixer with baffles at different impeller speeds are compared in Fig. 5. The shape of distribution curves approaches the differential distribution curves of an ideal mixer with increasing impeller speed.

The values of parameters a_i and b_i are presented in Table 1, with corresponding experimental conditions, as well as the sum of the squared differences between experimental and calculated response S_D and the mean residence time T_i. The mean residence time is the reciprocal value of parameter b_i. Values of the single parametric numerical dispersion model from paper [7] are also in Table 1. The values herein and from paper [7] are similar, but not identical because of the differences between the double and single parametric models. The mean residence time T_i is closer for high impeller speeds.

The disadvantage of the model designed lies in the fact that, while postulating the validity of Eq. (3) for the whole system as well as for individual parts of system, we assume in each case a constant Peclet number. The discrepancy in such an assertion lies in the fact that assuming a unique Peclet number in either of the mixer or the connector systems eliminates the existence of a unified Peclet number for the whole system. We estimated, therefore, the errors of the assumption of a constant Peclet number or mean RTD for the whole system. This was made on the basis of the Laplace transformation.

It can be readily shown (proof in [13]) that for the first two moments of the distribution we obtain:

$$T_i = \frac{1}{b_i} \quad [i = \mathrm{e}, \mathrm{m}, \mathrm{w}] \qquad (26)$$

$$\sigma_i^2 = \frac{2a_i - 2a_i^2(1 - \mathrm{e}^{-1/a_i})}{b_i^2} \quad [i = \mathrm{e}, \mathrm{m}, \mathrm{w}] \qquad (27)$$

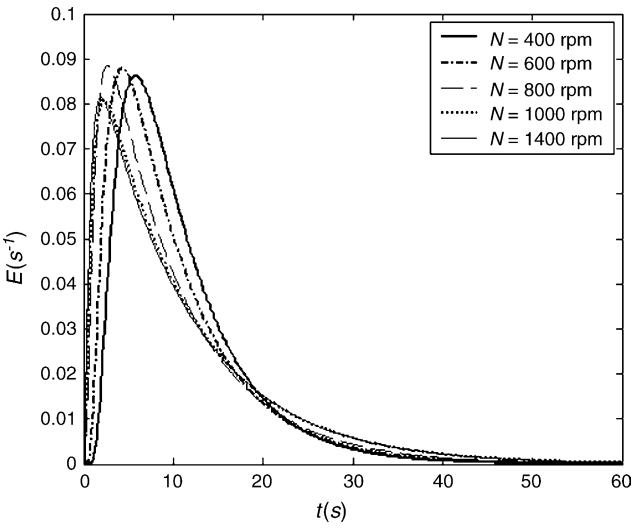

Fig. 5. Differential distribution function (m) effect of mixing agitation ($H = 3T$), $V = 2.74$ l, $Q_L = 6$ l/min.

And from Eq. (27) we obtain the relations:

$$T_w = 2T_m + T_e \equiv T_p \qquad (28)$$

$$\sigma_w^2 = 2\sigma_m^2 + \sigma_e^2 \equiv \sigma_p^2 \qquad (29)$$

Calculated values of these moments are presented in Table 2. The third column depicts deviations between them expressed as percentages. It is evident that when we compared calculated moments we found that deviations of first moment are up to 2.5% and deviations of the variance are up to 6%, which are acceptable from an engineering aspect. It should be duly noted, however, that values of T_w are always slightly higher and that assumption of constant values of a_w and b_w results in a systematic, albeit acceptable imprecision. In paper [14], the concentration dependency $c_w(t)$ from the twin system approach was approximated by a relation derived from the Laplace transform and it is shown

Table 1
Values of optimized parameters

N (min^{-1})	a_i	b_i (s^{-1})	S_i (s^{-2})	T_i (s)	$a_D = 1/Pe$	T_i (s)
Evaluation of external piping (i = e)						
	0.0281	0.06	1.55E−04	16.7		
Evaluation of experiment (i = w)						
600, without baffles	0.0222	0.0311	8.00E−07	32.2		
400	0.0367	0.0264	1.60E−04	37.8		
600	0.0458	0.0273	1.20E−04	36.6		
800	0.0600	0.0275	4.10E−03	36.4		
1000	0.0720	0.0260	1.55E−02	38.5		
1400	0.0750	0.0260	1.46E−02	38.5	Scargiali [7]	
Values of the one reactor (i = m)						
600, without baffles	0.1510	0.1280	1.60E−05	7.8		
400	0.2840	0.0919	1.50E−04	10.9	0.472	8.5
600	0.4730	0.0968	2.20E−04	10.3		
800	1.0100	0.0976	2.40E−04	10.2	1.111	10
1000	1.5200	0.0882	2.10E−04	11.3	1.587	11.1
1400	1.8600	0.0883	2.00E−04	11.3	2.564	11.6

Table 2
Values of first two moments of residence time distribution

N (min^{-1})	T_w (s)	T_p (s)	ΔT (%)	σ_w^2 (s^2)	σ_p^2 (s^2)	$\Delta\sigma^2$ (%)
600, without baffles	32.29	32.15	0.427	1089.18	1078.79	0.963
400	38.43	37.88	1.454	1589.38	1536.25	3.458
600	37.33	36.63	1.905	1526.43	1459.04	4.619
800	37.16	36.36	2.186	1550.78	1471.47	5.390
1000	39.34	38.46	2.290	1771.83	1676.97	5.657
1400	39.32	38.46	2.223	1777.04	1684.54	5.491

that we can do without the necessity of numerical approximation of convolution integrals. These calculations, however, have a high demand for computational precision.

5. Conclusions

The twin system is used to obtain solid residence time distribution in a two-phase stirred reactor. The dispersion model can be used for interpretation of residence time distribution curves of liquid or solid in continuous stirred vessels. The evaluation can be carried out as is described in paper [7] or by using Fourier's method of solution of dispersion model, which enables description of the twin system approach with the help of initial and boundary conditions. The dispersion model fits experimental response curves in both cases quite well.

The main difference between the procedure suggested in this contribution and the procedure presented in [7] lies in the fact that in the latter the "advection diffusion equation" used to delineate the axial dispersion model describing the system, is solved numerically by separate time stepping integration.

In this contribution, an analogous model is postulated by definition and only values of its two parameters are fitted, i.e. mean velocity of the particles (or mean residence time) and the dispersion coefficient (or Peclet number). This procedure is generally more accurate in the case of an aptly selected model.

The distribution curves with increasing impeller speed exhibit behavior close to that of the ideal mixer. When baffles are removed from the vessels, the mean residence time is significantly lower.

The error of not assuming constant overall coefficient was examined and evaluated as negligible for engineering purposes.

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